

# How many steps till I recover from a sudden lateral perturbation?

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## I. 1, 2, 3, 4, ... $N$ -Step Stability

**Goal:** Compute the set of lateral perturbations from which a human can recover by taking  $N$  or fewer steps.

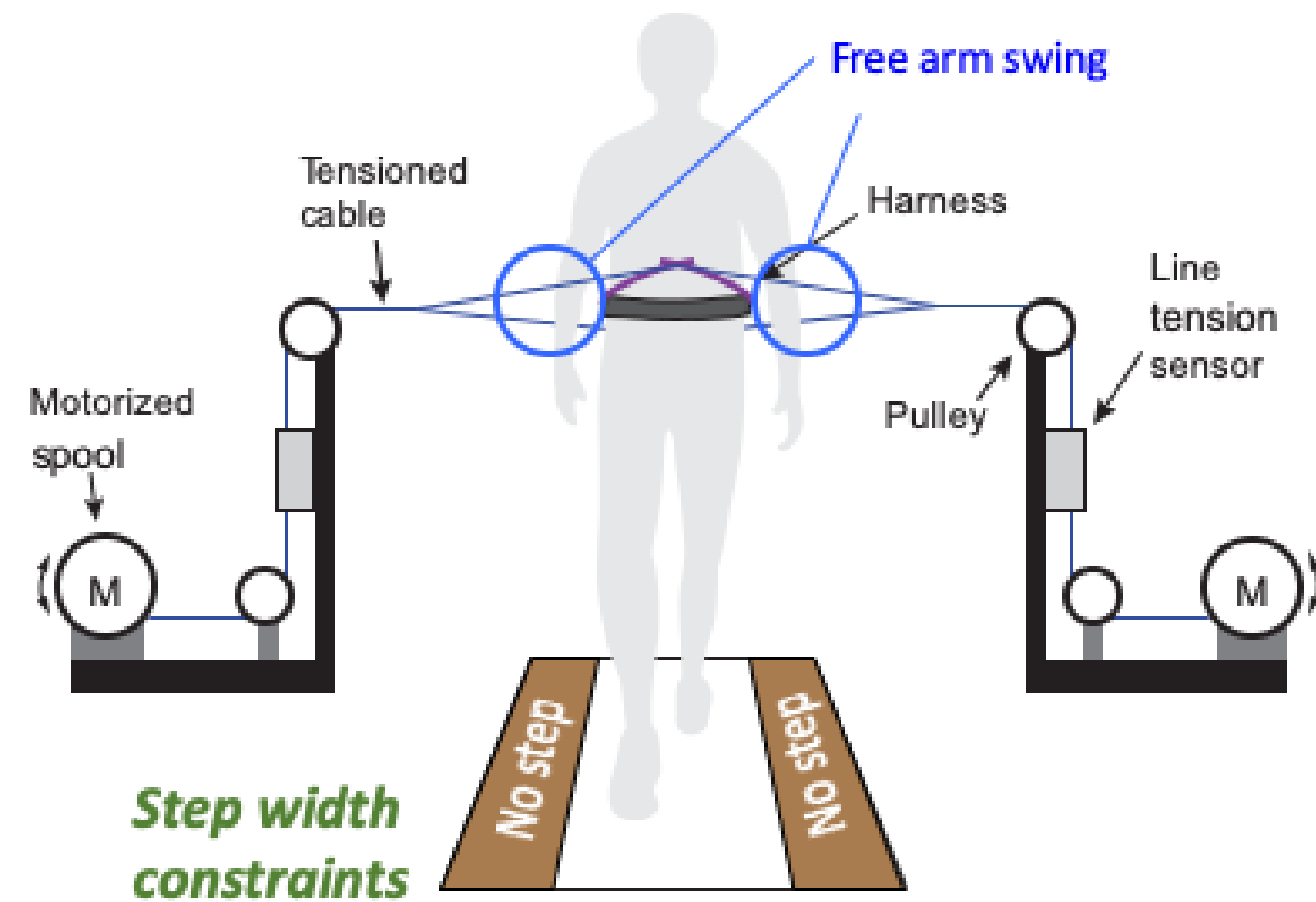
Characterizing human mediolateral stability could help:

- Diagnose high fall-risk individuals
- Improve prosthesis/exo design
- Guide rehabilitation
- Inform wearable robot control

## II. A Sudden Lateral Perturbation

Experiment: motor-driven cables pulled subjects laterally during walking:

- Motion capture, split-belt treadmill
- Varying magnitude, direction, timing
- Several speed and step width conditions
- 12 subjects; 5 female, 7 male (**only 1 subject analyzed here**)



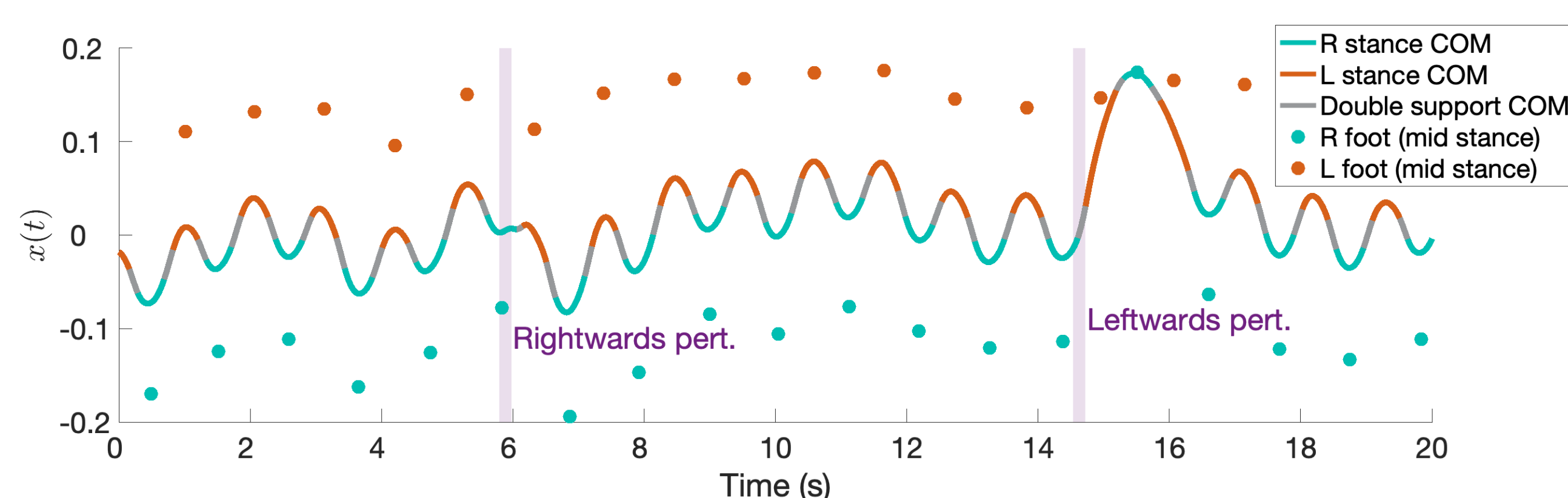
## III. I Model Walking with an LIP

### Experimental Data

- We model the **lateral** motion of the person's **Center of Mass (COM)**. At time  $t$ , let

$$x(t) := \text{lateral COM position, } \dot{x}(t) := \text{lateral COM velocity}$$

- A sample observed COM trajectory over the course of several steps is shown below:



### Model: Linear Inverted Pendulum with Reset Map

- Single support is modeled as a **Linear Inverted Pendulum (LIP)** with a foot:

$$\ddot{x}(t) = \frac{g}{z_c}(x(t) - \text{COP}(t))$$

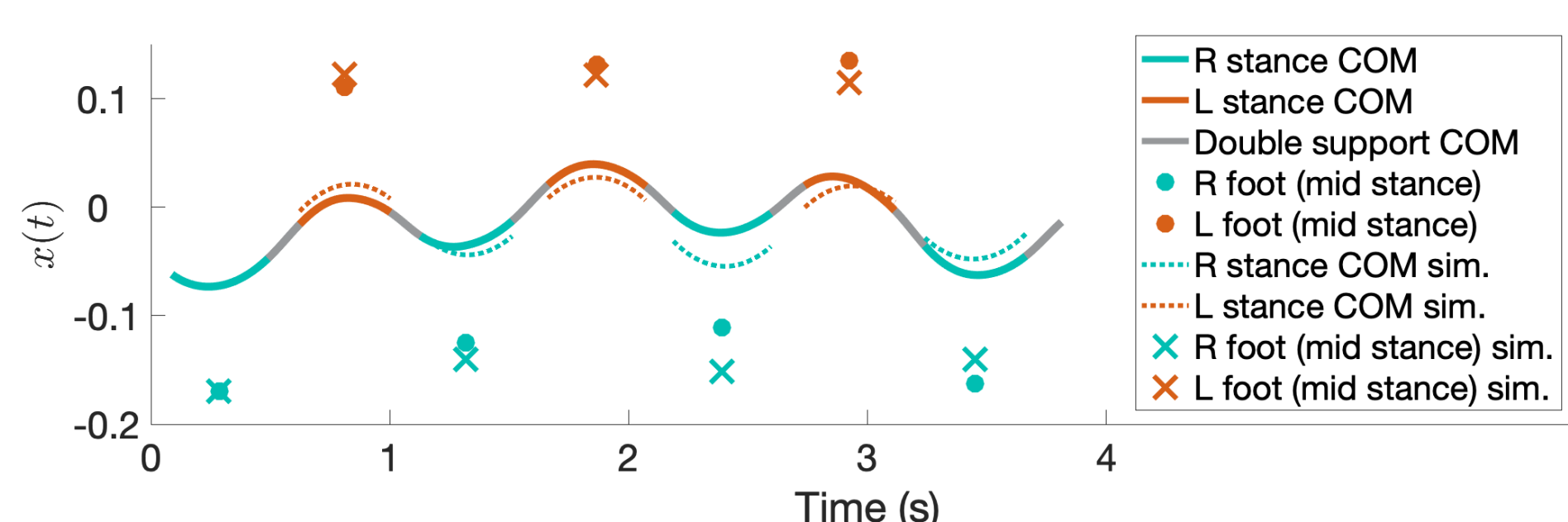
where  $\text{COP}(t)$  is the location of the Center of Pressure at time  $t$  inside the foot.

- Double support and foot placement are modeled using a linear reset map with uncertainty:

$$\begin{bmatrix} x(t^+) \\ \dot{x}(t^+) \end{bmatrix} \in \{A \begin{bmatrix} x(t^-) \\ \dot{x}(t^-) \end{bmatrix} + b\} + E$$

where  $A$  and  $b$  represent the linear map,  $t^-$  and  $t^+$  are the times of leading heel strike and trailing toe off, and  $E$  is a set bounding the error of the reset map.

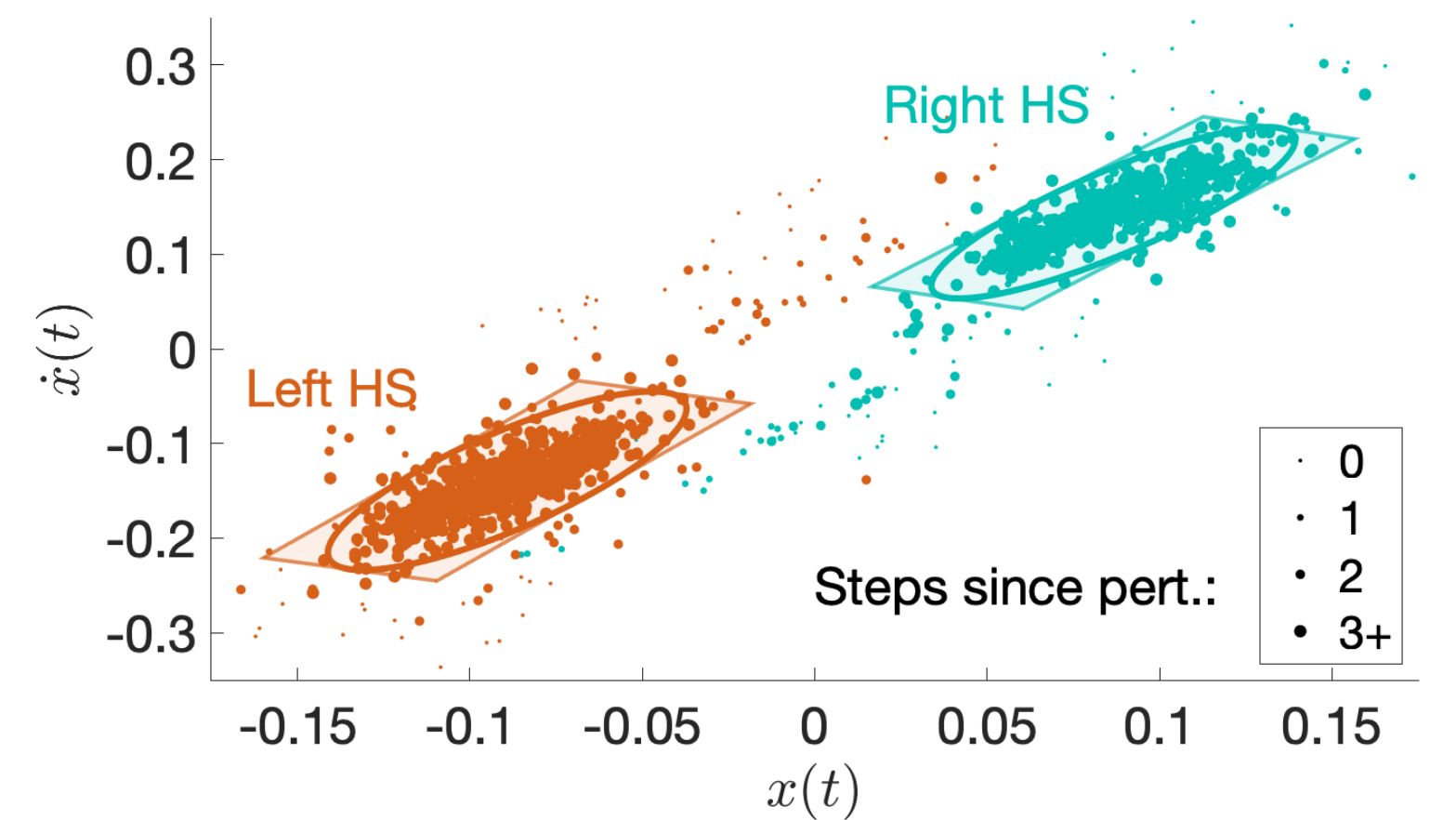
- A simulation (w/o uncertainty) is shown below, where the step timing is given:



## IV. A Basin Shows Stability

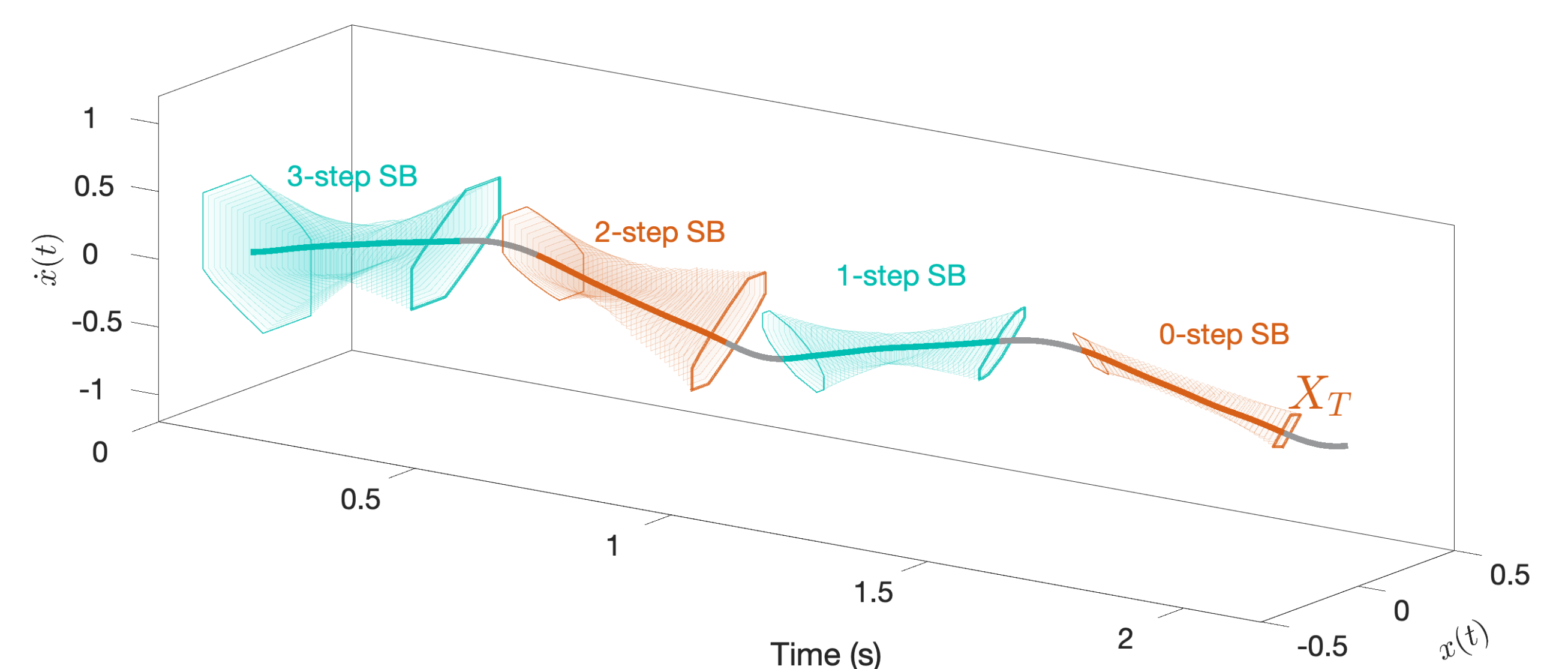
### This is my Nominal State

- A target set  $X_T$  at **heel strike (HS)** represents nominal walking.
- We form a Gaussian from all COM states at HS 3+ steps out from a perturbation.
- $X_T$  overapproximates the 95% confidence ellipse of this Gaussian.



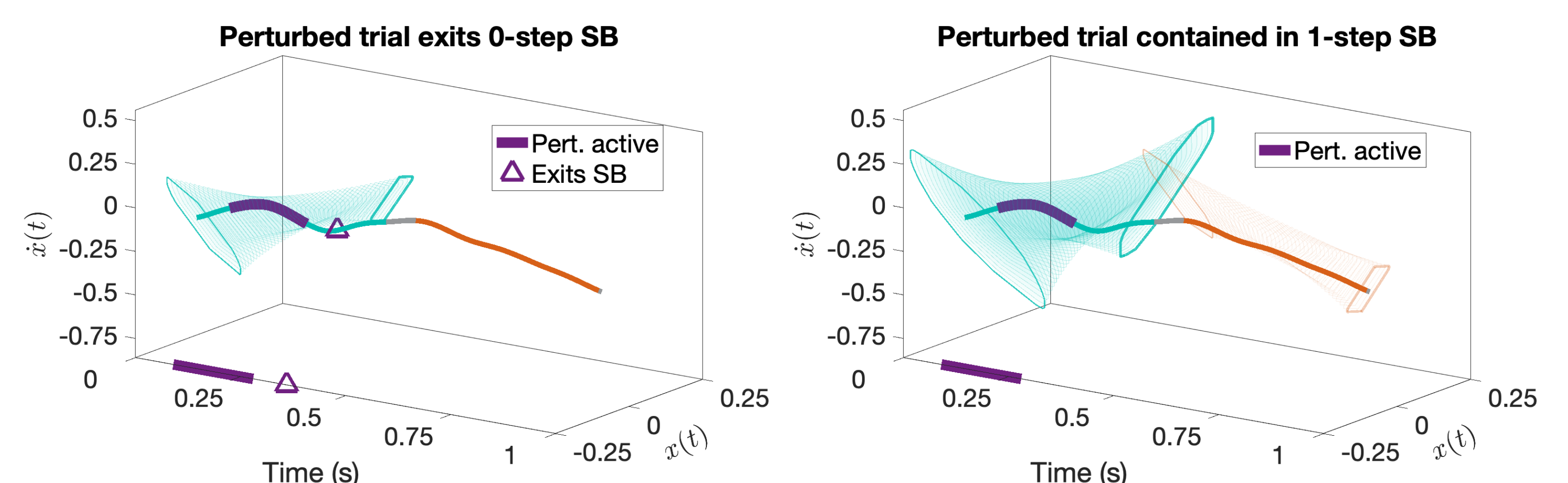
### Reachability

Inspired by capturability-based analyses<sup>1</sup>, we find the backwards reachable set of  $X_T$  under our model's dynamics after  $N$  steps, which we refer to as  $N$ -step **Stability Basins**. We use a reachability toolbox called CORA<sup>2</sup>, assume that step timing is given, and incorporate uncertainty at the step transitions.

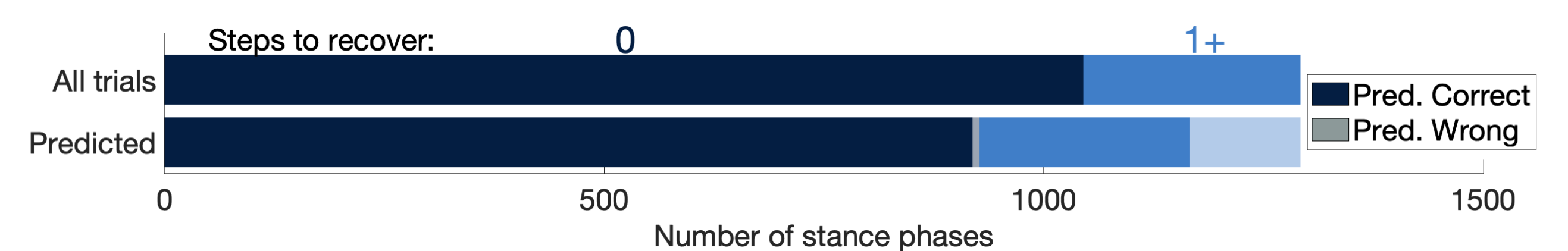


## V. ...Predicts How Long I'll Need to Walk Again?

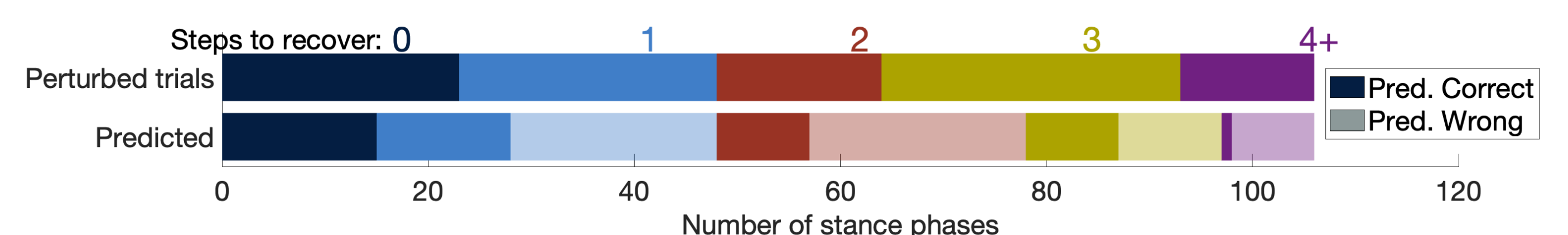
- We label each stance phase by the number of **steps to recover (STR)** until the COM state at HS is within  $X_T$ .
- The smallest Stability Basin fully containing a trajectory predicts the STR, illustrated below:



- The 0-step Stability Basin accurately distinguishes between 0 vs. 1+ STR:



- However, the prediction accuracy of 1 vs 2 vs 3 vs 4+ STR for perturbed trials is poor:



- Future work will focus on a more complex walking model that better models the dynamics of the 1+ STR trials.

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[1] Koolen, T., de Boer, T., Rebula, J., Goswami, A., Pratt, J. (2012). Capturability-based analysis and control of legged locomotion, Part 1: Theory and application to three simple gait models. The International Journal of Robotics Research, 31(9), 1094–1113.  
[2] M. Althoff, D. Grebenyuk and N. Kochdumper. Implementation of Taylor Models in CORA 2018. In Proc. of the 5th International Workshop on Applied Verification for Continuous and Hybrid Systems, pages 145–173, 2018.